$$\frac{[A_{\mathrm{TE}_{mn}}/A_{\mathrm{TM}_{mn}}]_1}{[A_{\mathrm{TE}_{mn}}/A_{\mathrm{TM}_{mn}}]_2} = \frac{[f_{c\mathrm{TM}}/f_{c\mathrm{TE}}]_1^2}{[f_{c\mathrm{TM}}/f_{c\mathrm{TE}}]_2^2} \frac{[f^2 - f^2_{c\mathrm{TM}}]_2}{[f^2 - f^2_{c\mathrm{TM}}]_1}. \quad (7)$$

Thus it has been shown that the relative mode amplitudes in the standard waveguide can be readily obtained once the relative amplitudes in the oversize waveguide have been determined.

In addition to the foregoing, we wish to note a correction and an omission. It has been pointed out by M. Sirel of Laboratorie Central Des Ponts Et Chaussees, that reference [11] should have referred to *Electronic and Radio Engineer*, not *Electronics*. Also, due to an oversight, acknowledgment was not made to J. F. Ramsey and Dr. P. A. McInnes, of AIL, for providing the theoretical and measured multimode antenna data used in this effort.

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Constructive Coupling in Directional Couplers

During the course of some diagnostic studies[1] of varactor harmonic generators, we discovered a surprising result using the experimental setup shown in Fig. 1. A signal source is connected through an isolator to port 1 of a 10-dB directional coupler, with the output load connected to port 4 through a slide-screw tuner. Port 2 is internally terminated, and port 3 has connected to it a slide-screw tuner and termination.

By simultaneously adjusting the two slide-screw tuners it was possible to maximize the power delivered to the output load. The reader is invited to pause at this point and estimate the amount of this maximum power for a 10-dB directional coupler. Would you believe a net insertion loss from the generator to the load of 10 dB, 7 dB, 3 dB? The answer, surprisingly, is approximately 3 dB.

This result can be proved using the scattering matrix formalism. For the circuit of Fig. 1 the following matrix equation holds

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & A & 0 & jB \\ A & 0 & jB & 0 \\ 0 & jB & 0 & A \\ jB & 0 & A & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ 0 \\ \Gamma_3 b_3 \\ \Gamma_4 b_4 \end{bmatrix} \quad (1)$$

where a_k and b_k are the incident and reflected waves on port k, and Γ_3 and Γ_4 are the voltage reflection coefficients of the tuner and load combinations at ports 3 and 4, respectively. B and A are taken to be real by proper

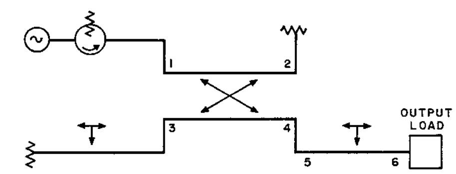

Fig. 1. Experimental setup using a directional coupler.

choice of reference planes. From (1) we have

$$b_3 = A\Gamma_4 b_4 \quad (2)$$

$$b_4 = jBa_1 + A\Gamma_3 b_3. \quad (3)$$

Substituting (2) into (3) gives

$$\frac{b_4}{a_1} = \frac{jB}{1 - A^2\Gamma_3\Gamma_4}. \quad (4)$$

Letting $\Gamma_3 = |\Gamma_3| e^{j\rho_3}$ and $\Gamma_4 = |\Gamma_4| e^{j\rho_4}$ in (4) and noting that $B^2 = 1 - A^2$ gives

both directions. Thus, half the power goes to port 1 and half to port 2, corresponding to a net 3 dB of insertion loss from port 6 to port 1. Application of reciprocity proves the desired result.

Thus we see that the unusual result occurs because of a resonance associated with the in-line arms of a directional coupler. This example shows that one must avoid such resonances in measurement schemes in order to maintain accuracy.

Such a resonance characterizes the behavior of the resonant ring circuit.[2] In the resonant ring circuit, neglecting losses, all of the power ends up in the termination on port 2. In the circuit of Fig. 1, as $A^2 \to 1$ (weak coupling) half of the generator power ends up in the output load connected to port 4, one-fourth of the power is delivered to the port 2 termination, and one-fourth of the power is reflected from port 1.

$$\frac{|b_4|^2}{|a_1|^2} = \frac{1 - A^2}{1 + A^4 |\Gamma_3|^2 |\Gamma_4|^2 - 2A^2 |\Gamma_3| \, |\Gamma_4| \cos(\rho_3 + \rho_4)}. \quad (5)$$

Since the tuner is lossless, the power absorbed in the output load is $|b_4|^2(1 - |\Gamma_4|^2)$, and the ratio of load power to generator power is

$$\frac{|b_4|^2(1 - |\Gamma_4|^2)}{|a_1|^2} = \frac{(1 - A^2)(1 - |\Gamma_4|^2)}{1 + A^4 |\Gamma_3|^2 |\Gamma_4|^2 - 2A^2 |\Gamma_3| \, |\Gamma_4| \cos(\rho_3 + \rho_4)}. \quad (6)$$

Since $A^2|\Gamma_3| \, |\Gamma_4|$ is positive and less than, or equal to, unity, this is a maximum for

$$\cos(\rho_3 + \rho_4) = 1$$

$$|\Gamma_3| = 1.$$

With these values, (6) becomes

$$\left. \frac{|b_4|^2(1 - |\Gamma_4|^2)}{|a_1|^2} \right|_{\max} = \frac{(1 - A^2)(1 - |\Gamma_4|^2)}{(1 - A^2 |\Gamma_4|)^2}. \quad (7)$$

When this expression is further maximized with respect to $|\Gamma_4|$, one obtains

$$|\Gamma_4|_{\mathrm{opt}} = A^2 \quad (8)$$

so that,

$$\left. \frac{|b_4|^2(1 - |\Gamma_4|^2)}{|a_1|^2} \right|_{\max} = \frac{1}{1 + A^2}. \quad (9)$$

As the coupling diminishes, $A^2 \to 1$ and the net insertion loss approaches 3 dB. Calculations for 3 dB, 10 dB, and 20 dB directional couplers give net minimum insertion losses of 1.76 dB, 2.79 dB, and 2.99 dB, respectively. Of course, circuit losses will increase these values somewhat, especially for the 20-dB coupler.

The analysis presented above does not lend much physical insight into the mechanism responsible for the unexpected results. A physical explanation is offered as follows. Consider the reciprocal situation where the source and load are interchanged. The two slide-screw tuner probes are inserted deeply so as to form a resonant cavity between them at the operating frequency. For a weakly coupled directional coupler, this resonant cavity has nearly equal waves traveling in

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[1] These studies concerned spurious oscillations resulting when the input circuit presents favorable impedances at two frequencies whose sum is the input frequency.

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[2] H. Golde, "Theory and measurement of Q in resonant ring circuits," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-8, pp. 560–564, September 1960.

Precision Design of Direct Coupled Filters

The direct coupled filter to be discussed in this correspondence consists of a length of transmission line with reflecting obstacles spaced at approximately half wavelength intervals. High-power capabilities can be achieved because the filter can be made without changing the size of the transmission line and, as will be described, without any special tuning devices. The following analysis is applicable for waveguides with inductive obstacles and also to coaxial lines with inductive posts. By duality it is also valid for coaxial or strip line with capacitive gaps.

For obstacles in rectangular waveguide, inductive posts were preferred to irises. The use of a programmed tape controlled milling machine is to be recommended for production runs of this class of filter. Each obstacle used is a pair of posts whose spacing along the guide and from each other can be carefully

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